Seat No. : _______________

ZM-141

May-2014

M.Sc. Sem.-II

407 : Statistics

(Reliability, Life testing & Bayer Estimation)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are of equal marks.

(2) Scientific calculator is permitted to use.

(3) Statistical table will be supplied on request.

1. (a) Define hazard function, bath tub failure function. Obtain expression for pdf as a function of hazard function. If the hazard function for the life time model is $h(t) = 3t^2/(343 - t^3)$; $0 < t < 7$, obtain corresponding life time model and reliability function.

OR

Define cumulative hazard function. Let X and Y are two independent life time random variables having different life time distribution $f_x(x)$and $f_y(y)$ respectively. Let $W = \min(x, y)$, show that the hazard function of W is equal to the sum of the hazard functions of x and y. Is it true for cumulative hazard function ? Justify your answer.

(b) Suppose the lifetime random variable x has exponential distribution with mean θ, $\theta > 0$. Show that (in usual notations)

(i) $R(t_1 + t_2) = R(t_1)\, R(t_2)$

(ii) $E[x - c \mid x \geq c] = E(x)$, when C is constant

OR

Define mtbf(m). Show that if R(t) is reliability function for a device then mtbt is given by $m = \int_0^\infty R(t)\, dt$. If $R(t) = e^{-\left(at + \frac{bt^2}{2}\right)}$, $t > 0$, obtain its mtbf.

2. (a) Show that the life time distribution is exponential if and only if the distribution possess memoryless property, in case of continuous life time model.

OR

Let $T_1, T_2, ..., T_n$ are iid exp(λ); $\lambda > 0$. The time between $(k-1)^{th}$ and k^{th} failure is defined as $G_k = T_{(k)} - T_{(k-1)}$, k = 1, 2, ..., n with $T_{(0)} = 0$. Show that GK's are independent exponential variates but not identically distributed. Compare the hazard rates of T_k and G_k.

(b) Obtain MLE and UMVUE of the reliability function in case of exponential model with mean $\theta > 0$, based on Type – II censored sample.

OR

For exponential life time model with mean $\theta > 0$, obtain MLE of θ, expected total test time and expected test termination time under type – II censoring WOR and WR. Compare your answers under both the schemes and give your comments.

3. (a) For Rayleigh lifetime model obtain MLE of hazard function based on Type-II censored sample.

OR

Let K components having iid exponential life time model with mean life time $\theta > 0$ are connected in (a) series and (b) parallel systems. Obtain life time distribution and mean life time of both the systems.

(b) The reliability of a communication channel is 0.40. How many channels should be placed in (i) parallel and (ii) series systems so as to achieve at least 0.8 system reliability ? Which system do you prefer ? Why ?

OR

Obtain reliability of series and series-parallel systems. Give real life examples for both the systems. If reliability of components is $e^{-t/\theta}$ at time t and all the components are iid then obtain life time distribution of both the systems.

4. (a) Define prior distribution, posterior distribution, risk function, Bayer risk and Bayer estimator. Discuss a real life situation where Bayer estimation can be apply.

OR

Define squared error loss and weighted squared error loss function. Obtain general form of Bayer estimator of some function $\psi(\theta)$ under both the loss functions.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from Bernoulli distribution with parameter p, $0 < p < 1$. If the prior distribution of p is uniform $\cup(0, 1)$, obtain Bayer estimator of $\frac{p}{1-p}$ in case of (i) squared error loss function, (ii) weighted squared error loss function with weight $w(p) = p(1 - p)$.

OR

Let $X_1, X_2, \ldots, X_n$ be a random sample from exponential distribution with mean $\frac{1}{\theta}$, $\theta > 0$. The prior distribution of θ is also exponential with mean β, $\beta > 0$. Obtain Bayer estimate of $R(t) = e^{-t/\theta}$ under (i) squared error loss function and (ii) weighted squared error loss function with weight $w(\theta) = e^{\theta}$, $\theta > 0$.

5. Answer the following :

(1) Define reliability.

(2) State the distribution having bath-tub hazard function.

(3) State the expression for reliability at time t_i for ungrouped data under non-parametric method.

(4) Say T/F :

If the death rate of a person that smoker is twice that of a non-smoker at each age, then non-smoker has twice the probability of surviving a given number of years as does a smoker of the same age.

(5) For any continuous life time random variable, its cumulative hazard function always follow ______ distribution.

(6) State self-reproductive property of exponential life time model.

(7) Let $X_1, X_2, \ldots, X_n$ are iid exponential variates with mean $\frac{1}{\lambda}$, $\lambda > 0$. Define $Z_i = (n - i + 1)$ $(\lambda_{(i)} - \lambda_{(i-1)})$, $X_{(0)} = 0$; $i = 1, 2, \ldots, n$. State the distribution of $2\lambda \sum_{i=1}^{n} Z_i$.

(8) Let 10 items are put on a test under with replacement. The failure time distribution is exponential with mean 5 hrs. What is the expected number of failures during first two hours of the test ?

(9) State 2-parameter exponential life time model.

(10) Say T/F :

Under Type-I censoring WOR, the MLE of mean life time of exponential life time model with mean $\theta > 0$ is unbiased for θ.

(11) If failure items are not replaced during the test, state the distribution of number of failures during the time interval (0, t).

(12) If the hazard function is $2\lambda t$, $t > 0$, $\lambda > 0$, state corresponding cumulative hazard function.

(13) Say T/F :

Normal distribution is the proper prior distribution for the mean of normal distribution.

(14) Say T/F :

If MLE of a parameter θ exists, it is always same as Bayer estimator of θ under squared error loss function.
